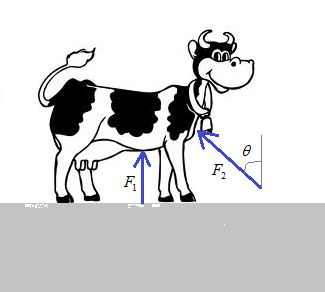
**Question 1 (5 points)**

If **F**1 has a magnitude of 200N, and **F**2 has a magnitude of 100N, is it necessarily true or not that the magnitude of the net force is 300N?

No.

**Question 2 (15 points)**

Two people push on a cow, as shown below. **F**1 has a magnitude of 400N and is directed vertically. **F**2 has a magnitude of 700N and is directed at an angle θ = 25◦ from the vertical. What is the magnitude and direction of the net force acting on the cow



Net force is:



So magnitude is:



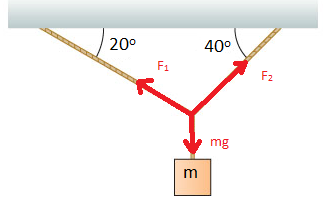
**1**. A force of 1500N acts on a car with mass m = 2000kg. How long will it take for the car to reach a speed of v = 20m/s?



And,



**Question 2**. A m = 5kg block hangs from the two ropes below. What is the tension in the right rope?



The forces acting on the knot are displayed. They must add to zero in both directions since the knot is not (ha ha) moving. So we have:



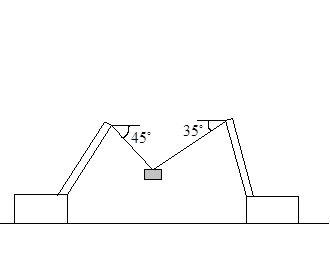
and in the y-direction:



Tension in other rope is, F­1 = 43.2 N.

**Problem 3.**

Two cranes are being used to pick up a car. What is the tension in the two cables? You can take the mass of the car to be m = 1500kg.



Let the left tension be F1 and the right be F2. Then N2L reads:



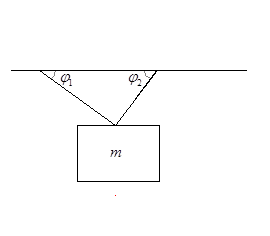
and,



and the other,



**2**. Find the tension in the strings, supposing that the sign has a mass m = 15kg. The angles are φ1 = 25◦ and φ2 = 55◦.



Adding forces in x-direction:



and in the y-direction:



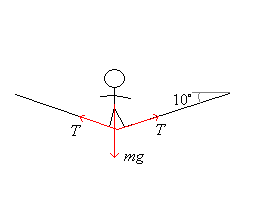
and going back to F1,



**Problem**

A high-wire artist stands on a rope, bent at a 10 degree angle. If his mass is 50kg, what is the tension in the rope?

Picture is below,



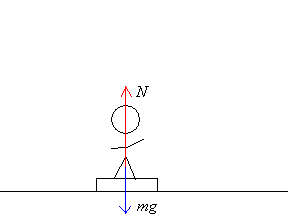
We can calculate the tension by summing up the forces in the x and y directions (though we’ll only need the y direction sum as you’ll see). Also, we specify the acceleration to be 0, since presumably, Ann, is stationary on the rope (hopefully).



So the tension is T = 1411 N.

P 11. A person jumps up and then down on a scale. When landing the scale reads 75% of his weight. What is his acceleration?

A picture is shown below,



We can find the acceleration using N2L. They say, by the way, that N (which would be the reading on the scale) is only 0.75 the person’s weight (mg). So we have,



so the acceleration is downward (because ay is negative) and has a magnitude of 2.45m/s2.

**3**. A hockey puck slides across the ice with initial velocity v = 25m/s. If it comes to rest in 7s, what is the coefficient of kinetic friction between the ice and puck?

Adding forces in x-direction we have:



To get the normal force we must use N2L in the y-direction:



And the acceleration is given by:



Plugging these in



**Problem**

Consider baseball player sliding into homebase – want to calculate the point where he has to begin sliding (at velocity v) to hit homebase d away.

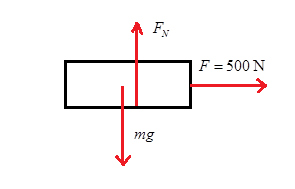


So,



**1**. A force of 500N acts on a car with mass m = 1000kg. How long will it take for the car to reach a speed of v = 20m/s?

Force diagram is given below:



Newton’s second law (N2L) reads:

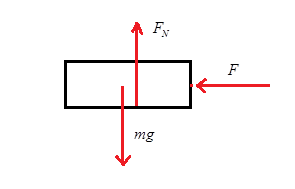


Note that we don’t really need to use the results of N2L in the y-direction. The amount of time it will take to reach 20m/s can be obtained from kinematics equations,



2. The same truck (m = 1000kg) as above is rolling down the highway with an initial speed of v = 20m/s. What force would be required to stop the car within a distance of 30m?

Force diagram is shown below:



N2L reads:



Again the results of N2L in y-direction are not important here. Now, in order to get F we need to know *a*. We can get *a* from a kinematics equation. Now we want to slow the car, initially going 20m/s, to rest, in a distance of 30m. Let’s fill this information into the vx equation:



OK we still have an unknown, namely t. We only have one equation left, the x-equation so that better help.



We have two unknowns in the x-equation too, which is unfortunate. But we can solve for t in the first equation, plug it into the second equation, and solve for F (the standard procedure for solving two equations with two unknowns). So solving for t in first equation yields,

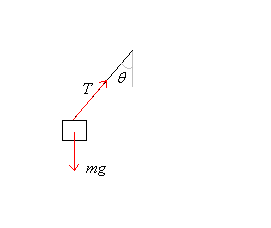


and plugging this into the second equation yields:



P16. You hang keys from a string and notice that they make an angle when you accelerate from 0 to 28m/s in 6s. What is this angle?

Consider the free body diagram,



If your car is accelerating at a rate of:



(and implicity ay = 0 since the car isn’t moving vertically), then the fuzzy thing must be accelerating at the same rate, since it is attached to the car. Therefore, according to N2L, we have,



and in the y-directoin we have,



To get θ, we can solve for T in the y-equation,



and plug it into the x-equation



taking the inverse tangent of both sides,



**Problem 1.**

In the preview to Winter Soldier, some dude with a mask catches Captain America’s shield. Suppose the shield has a mass of 10kg and was thrown at 25m/s (about 55mph). And suppose this guy catches it, thereby slowing it down to a speed of 0m/s in the span of maybe 5cm. What force must this guy have exerted?

The force he exerted, i.e. the force necessary to decelerate the shield from 25m/s to 0m/s in the span of 3cm, can be calculated from N2L: F = ma. m = 10kg obviously, and we need to get *a*. We can do this with the two kinematics equations. x = x0 + v0t + (1/2)axt2 → 0.05 = 0 + 25(t) + (1/2)axt2→ 0.05 = 25t + (1/2)axt2. And vx = v0x + axt → 0 = 25 + axt → t = -25/ax. Plugging this into the first we get:

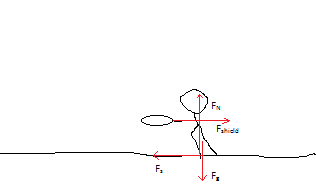
0.05 = 25[-25/ax] + (1/2)ax[-25/ax]2 → 0.05 = - 252/ax + (1/2)∙252/ax → 0.05 = -252/2ax →

ax = -252/(2∙0.05) = -6 250 m/s2. So then the force required is: F = ma = (10kg)(-6 250 m/s2) = -62 500 N. Note the (-) just means the direction of the force is to the left. And for comparison’s sake, this force would be about 14 000 lbs.

**Problem 2.**

When he catches it, he doesn’t even move backwards. What coefficient of static friction between his feet and the ground would be necessary for this to happen? On the other hand, if we assume that he would indeed move backwards, and the coefficient of kinetic friction is μk = 0.58, what would be his acceleration? Take his mass to be m = 85kg.

A depiction of forces would look like this:



To not move we would have:



We can solve for FN with N2L in y direction:



Plugging this back into our previous equation:



So surface must be coated with super glue. On other hand, it being more likely that he would move since μs likely wasn’t this big, his acceleration would be given by N2L,



**Question 1.** A baseball (m = 0.20kg) strikes your head at a speed of 30m/s, coming to rest in 0.0015s. What force does it exert on your head?

The acceleration is given by:

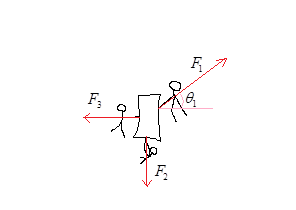


and so the force on the baseball, and by N3L the force on your head, is:



**Problem 3**

Three people are fighting for a dress on Black Friday. Assuming the dress is stationary, what force (magnitude and direction) is the first person exerting? Let F2 = 100N, F3 = 75N.



We will use N2L. Let x be direction to right, and y be direction up. Then,



and,



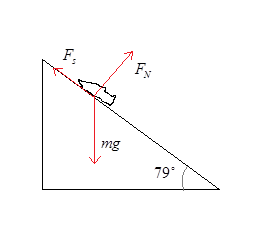
Let’s solve the top equation for F1 and plug it into the second equation to get θ1. So F1cosθ1 = 75 → F1 = 75/cosθ1. And therefore F1sinθ1 = 100 → [75/cosθ1]sinθ1 = 100 → 75tanθ1 = 100 → tanθ1 = 100/75 → θ1 = tan-1(100/75) = 53.1◦.

Now plugging this angle into F1 = 75/cosθ1 we get F1 = 75/cos(53.1) = 125 N.

**Problem 4.**

You wish to measure the coefficient of static friction between two surfaces, say a shoe and some synthetic surface. To do this, you place the shoe on the surface and slowly raise the angle of inclination of the surface until the shoe starts to slide. If the shoe starts to slide at an angle of 79◦, what is μs?

Situation looks like this:



Orienting our axes so that x points up the plane (along Fs) and y points along FN, we have:



To get FN we use N2L in y direction:



Plugging this in we get:



6. The floor of a railroad flatcar is loaded with loose crates having a coefficient of static friction of 0.4 with the floor. If the train is initially moving at a speed of 60 km/h, in how short a distance can the train be stopped at constant acceleration without causing the crates to slide over the floor?

Using N2L,

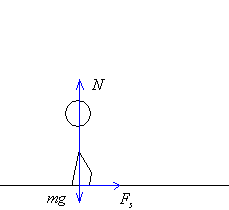


so this is the maximum acceleration. And so the shortest distance is:



**P17**. You’re standing on a train which begins to accelerate at rate a = 0.2g m/s2 to the right. What is the minimum necessary μs to keep you from slipping?

Draw a free body diagram of yourself,



The train is accelerating to the right, which will tend to cause you to slip along the floor, leftward. But the (static) friction force between your feet and the floor will oppose this tendency and exert a force to the right. Now in order for your acceleration to match that of the train, we must have,



If we want to know the minimum μs necessary to give you this acceleration, then we let Fs be its maximum value, Fs = μsN, and so,



to get N, we use N2L in the y-direction,



so filling this in…



**Question 10**. Suppose you’re standing on an ice rink, when you decide to run after someone skating by. How quickly can you accelerate if your mass is 60kg, and the coefficient of static fraction between your shoes and the ice is 0.15?

We just use N2L. The x-force on you is friction, and the y-force is just N and mg. So we have,



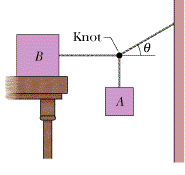
Now plug the y-direction expression into the x-direction one:



which is,



7. Block A weighs 150N and block B weighs 200N. And suppose θ = 30 degrees. What is the minimum coefficient of static friction necessary between the table and block B in order for the blocks to remain stationary?



Assuming that block B is stationary, let’s find the tension in the horizontal rope. Applying N2L to the knot connecting the ropes, and letting T1 denote the tension in the horizontal rope, and T2 the tension in the angled rope we have:

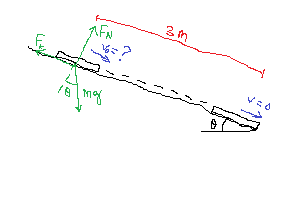


Now the friction force must be able to counteract this force. So we must have:



**Question 4.** You and your friend Peter are putting new shingles on a roof pitched at 20° . You're sitting on the very top of the roof when Peter, who is at the edge of the roof directly below you, 3m away, asks you for the box of nails. Rather than carry the box of nails down to Peter, you decide to give the box a push and have it slide down to him. If the coefficient of kinetic friction between the box and the roof is 0.45, with what speed should you push the box to have it gently come to rest right at the edge of the roof?

Force diagram looks like:



N2L in the y direction yields:



N2L along the x direction yields:



Then the velocity of the shingle as a function of time is:



Since it will come to rest at 3m away, vx = 0 → t = v0/0.79. The position as a function of time is:

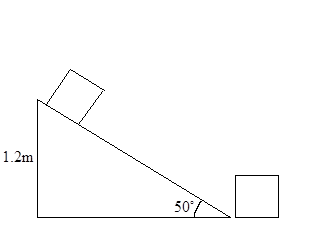


Setting x = 3, substituting t = v0/0.79 and solving for v0 yields:



**Problem 4.**

In the process of moving to a new apartment, you notice a box slides down a 1.2m tall ramp in 3s. What is the coefficient of kinetic friction between the box and ramp? It’s pretty big.



Pointing our x-axis down the plane, and y-axis perpendicular to it, we have according to N2L:



and,



Plugging this into the top equation we get:



Now the length of the ramp is x = 1.2/sin(50◦) = 1.57m. And so then according to the kinematics equation we have:

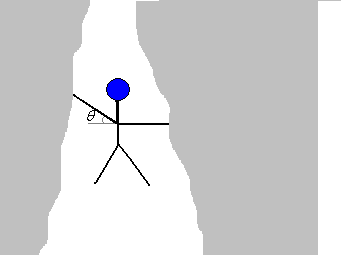


Equating these two we have:

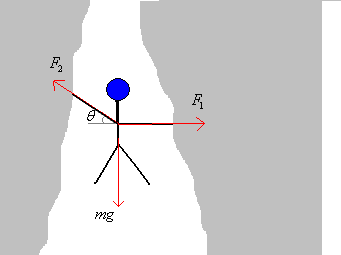


**Example: Rock climber**

Suppose a rock climber (m = 60kg) is holding onto two ledges. If the angle θ = 25° what forces must he be exerting on the ledge with his arms?



To determine this, we draw a free-body diagram labeling all the forces acting on the person. So we’d have the two forces exerted by his arms, and also gravity.



Now since the person (hopefully) is not going anywhere, and therefore definitely not accelerating, we have according to N2L,

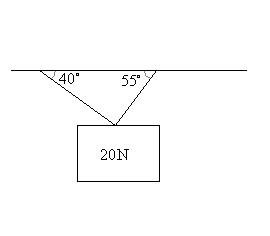


Therefore,



Is it feasible for the person to exert these forces? Multiplying by 0.225 to convert N to lbs, we’d have F1 ~ 250lbs, and F2 ~ 280lbs. These forces would likely be too great to sustain for long.

4. Find the tension in the strings – supposing that the sign weighs 20N.



Let F1 be tension in left string, and F2 be tension in right string. Then adding up forces in x-direction we have:



and adding up forces in y-direction gives:



Plugging top equation into the bottom one gives:

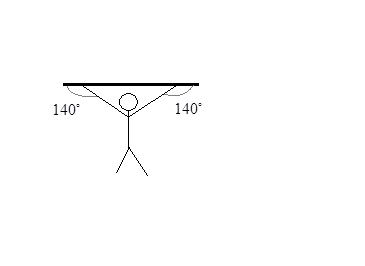


from which we get:

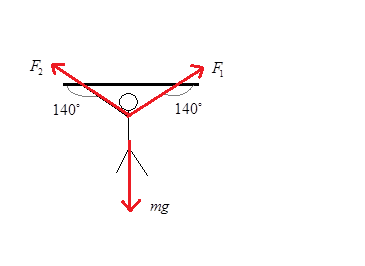


**Problem 4**

Suppose you’re performing a wide-grip pull-up. Your arm lengths are exaggerated a bit for clarity. If your mass is m = 70kg, what minimum force must you exert with each arm to pull yourself up – this would be the force required to just hold yourself in that position. Just for comparison’s sake, calculate the fraction of your weight (W = mg) that this required force equals.



Drawing the forces…



Note that the angle the F’s make with the horizontal is 40 degrees. So then filling these into N2L:



and in the y-direction:



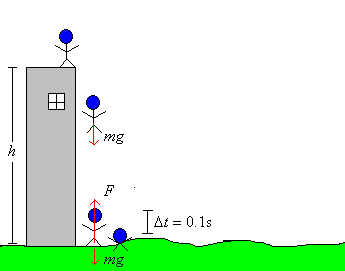
We can use the top equation F1 = F2 to substitute into the bottom equation and write:



and F1 = 533 N as well, since it equals F2. The fraction of weight this force represents is F/W = 533/mg = 533/[70∙9.8] = 0.78 = 78%

**Example: Falling from the top of a building**

Suppose you fall from a 28.5m ≈ 90ft. building. And the ground brings you to rest in about 0.1s. What force does the ground exert on you then? Assume the mass is 70kg.



The solution proceeds in two stages. First we determine our velocity at the bottom of the building right before we hit the ground. And then we use the fact that F, with mg, must slow us down from that velocity to 0 in a span of 0.1s. For part 1, the force acting on us is mg, and therefore our acceleration will be 9.8m/s2 downward. So then solving for vy at the bottom,



Note that vy is actually negative since it is going down, and so we have,



Now our acceleration between first coming in contact with the ground, and coming to rest there is:



The forces acting on us during this time are , and so according to N2L,



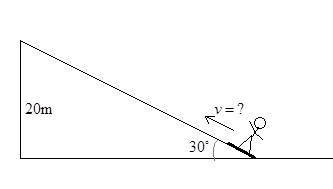
Filling in the values, we get,



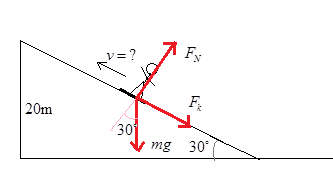
So falling off of a 28.5m ≈ 94ft building, you would experience around the same force that you would in a crash at 50mph (without a seatbelt). So that puts the crash in perspective.

**Problem 5**

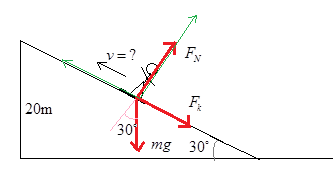
Suppose you’re skiing up a 20m tall slope at the indicated angle of inclination. What minimum initial speed must you have at the bottom to just make it to the top? Let the coefficient of friction between the snow and your skis be μ­k = 0.09.



Drawing the forces…



Orienting our axes as shown below:



and filling the forces into N2L we have:



and in the y-direction:



Now that we know the normal force, we will plug it into the top equation and solve for ax. We get:



Now we need to determine the required initial speed to just barely make it to the top. If he just barely makes it, then his velocity at the top will be 0. So we need to figure the required initial speed so that his final speed at the top will be 0. We’ll use the two x-equations for this. To start we will need to know the x-distance traveled and this is given by xsin30 = 20 → x = 20/sin30 = 40m. And now then,



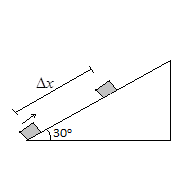
and the x-velocity equation yields:



Plugging this time into the top equation and solving for v we get:



**Question 5**. A 3kg wood block is launched up a wooden ramp that is inclined at an angle of 30°. The block’s initial speed is 7m/s. The coefficient of kinetic friction between the block and ramp is μk = 0.22. How far up the ramp, Δx, does the block slide?



Acceleration of block is given by:



N2L in y-direction gives us FN:



Plugging FN into the N2L x-direction gives us:



And now we can get the distance up the incline the block travels. Filling this into the x-equation we get:



Filling our information into the vx equation we have:

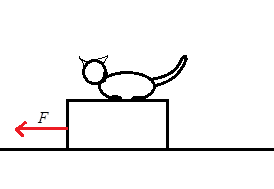


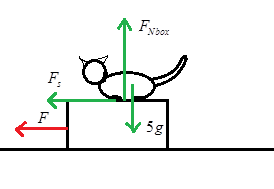
Plugging this time into the x-equation we have:



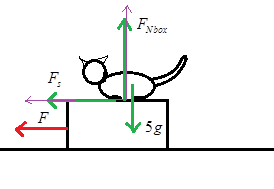
**Problem 6**

Suppose your cat is sitting on a box, which lies on a *frictionless* floor. If you pull the box with a small force, then the cat will move with the box. If you pull the box with a large force however, you can pull the box right out from underneath the cat. Let the mass of the box be mbox = 0.7 kg, and let the mass of the cat be mcat = 5kg. Let the coefficient of static friction between the cat and the box be μs = 0.96. What is the largest force F that you can apply to the box without the cat slipping? I would like you to solve the problem in this order. (a) Draw the forces acting on the cat – there are three. (b) Draw the forces on the box – there are four (consider Newton’s third law to get the fourth) (c) Apply N2L in the x, y directions to the cat and box separately, giving them each the same acceleration in the x direction. And you may assume that the static friction force is at its maximum value since the cat is just about to slip. (d) Solve for F.





For convenience, I’ll have x axis point left and y axis point up, like this (displayed in purple):



Then filling in N2L for the cat we have:



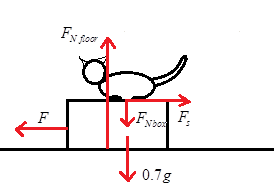
FN box is just the normal force that the box exerts on the cat. Note that I have set Fs = Fsmax (which is equal to μsFN box) because I am looking for maximum force with which you can pull to have cat move with box, and this maximum force will correspond to maximum Fs. To determine FN box we go to the y-direction:



Filling this back into the top equation we get:



Now we will draw the forces on the box,



Note that FNbox and Fs were forces that the box exerted on the cat, and by Newton’s third law, these forces will be exerted on the box, in the opposite direction. And you will observe that the arrows representing these forces are indeed pointing in the opposite direction to that drawn on the cat. Now we can fill these forces into N2L.

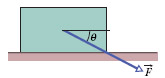


**Example**

Suppose you have a sheet of mass m = 0.15kg, and a plate on top with mass m = 0.90kg. The coefficient of static friction between them is μs = 0.7. What is the minimum force you must apply to get the sheet to slide underneath the box?

Well, we’ll look at the maximum force we can apply to get them to move together. So F = ma = (0.15 + 0.90)a = 1.05a. And maximum acceleration of box is a = F­s/m = μsmg/m = μsg. And so F = 1.05(μsg) = 7.72N.

5. A 5 kg block is pushed along a horizontal floor by a force **F** of magnitude 30 N at a downward angle *θ* = 40°. The coefficient of kinetic friction between the block and the floor is 0.25. What the magnitude of the block's acceleration?



Adding up forces in x-direction we have:



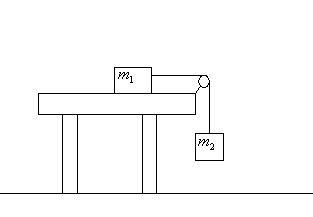
In y-direction we have:



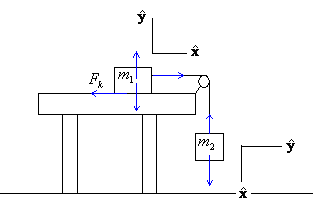
Plugging this normal force into the top equation gives…



10. Suppose that μk = 0.2, m1 = 10kg, m2 = 20kg. What then is the acceleration of the two blocks? What is the tension in the rope? What is the minimum coefficient of static friction that would be required to hold the blocks steady?



Labeling the forces…



And applying N2L to the first block,



The y-equation implies,



and the x-equation implies,



now applying N2L to block 2,



Now to solve for a, we’ll solve for T in the first equation,



and then plug into the second,



plugging in the numbers,



The tension in the rope is:



To determine the Fs necessary to hold the blocks still, we assume the blocks are still as a result of Fs, and then calculate what μs must be. We can set Fs = μsN since we’re looking for the minimum necessary μs. Starting out with block 1 we have,



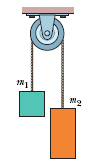
Plugging this last y-equation into the x-equation we get T = μsm1g. But we still cannot determine μs yet since we don’t know what T is. So we look to the second block.



Using this expression for T we can say:



6. The figure shows two blocks connected by a cord (of negligible mass) that passes over a frictionless pulley (also of negligible mass). The arrangement is known as Atwood's machine. Block 1 has mass *m*1 = 2 kg; block 2 has mass *m*2 = 4 kg. What is the tension in the cord?



Orienting our x-axis upwards for m1 and downwards for m2 – to keep the x-axes following the direction of motion, we have for m1:



and for m2,



Solving for ax in the first equation and plugging into the second we have:

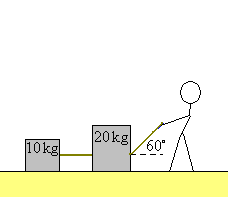


Plugging in the masses we get:

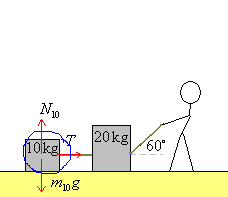


**Example: Pulling on two blocks**

A person pulls two blocks with a fore of F = 200N. Determine the acceleration of the blocks.

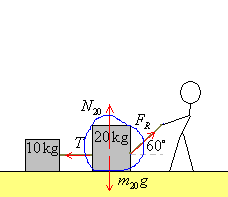


Like before, we can either analyze the two blocks individually or together as a whole. Let’s do it individually first. Analyzing the left block we have:





And the right block

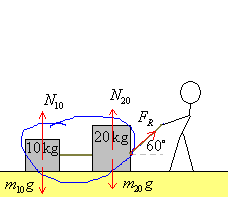




Solving for T with block 1 equation and plugging into the block two equation we get:



But of course its easier to treat the blocks as a whole. Then we have:



and writing out N2L gives:

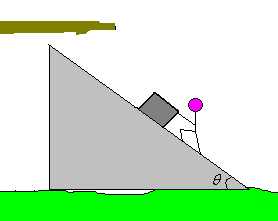


which immediately gives:

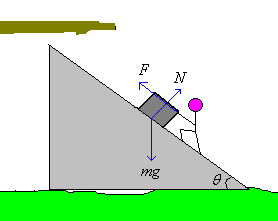


**Example: Holding block on incline**

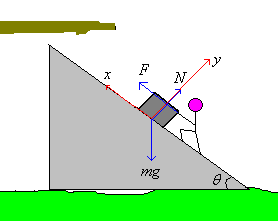
Suppose we try to move a piano up an inclined plane. If the incline is θ = 30° for instance, and the mass of the piano is 300kg, what force do we exert?



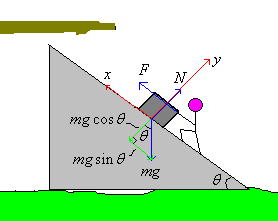
Again, let us draw the forces acting on the piano. There will be the force applied by the person – which we’ll assume is inclined along the surface of the plane. Gravity will act on the piano as before. Additionally, there will be a force exerted by the surface of the plane on the piano. This force is called the *normal* force – because it acts *normal* (perpendicular) to the surface. If the surface is completely smooth, then there will be no friction. We’ll make this unrealistic assumption in this case – next lecture we’ll talk about friction. Labeling all the forces, we have,



Now before we proceed with N2L, we will in hindsight find it convenient to work with a set of axes oriented differently than usual – we’ll orient the x-axes along the plane in the direction of motion and the y-axis perpendicular to it.



Now let’s break the forces into their components along these axes. Conveniently, F and N are along the axes already – only mg must be decomposed (which is one of the advantages of these axes).



Now then finally, if we assume that we’re moving the piano up with constant velocity, then the sum of the forces acting on the piano must add to 0. Proceeding then with N2L,



This breaks down into,



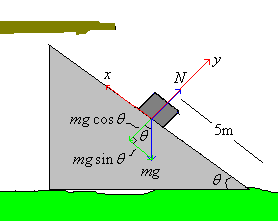
But we only care about the left one.



**Example: block sliding down incline**

If we let the block slide down the incline plane, from a height of 5m. How fast will it be going by the time it reaches the bottom?

The forces acting on the crate are shown below:



Applying N2L to get the acceleration, we get:



Equating x-components we find the acceleration to be:

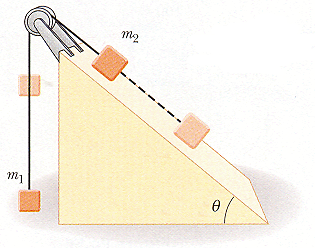


So after falling through a distance of 5m with that acceleration, its speed will be:

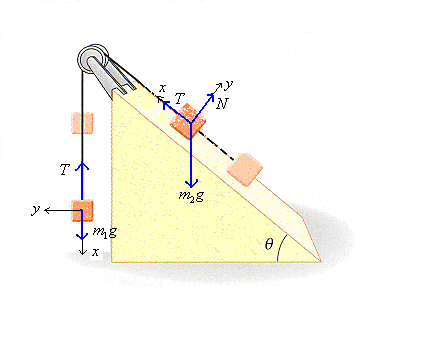


**Example: Two coupled blocks on inclined plane**

Consider two masses attached to an inclined. Suppose m1 = 10kg, and m2 = 5kg, and that the plane is inclined 30 degrees. What is the acceleration of the blocks?



To determine the acceleration we look at the forces acting on each object, and we orient our x-y axis for each block so that the x-axis points along the plane of motion of the objects.



Next we write out N2L for each object. For block 1 we have,



and for block 2 we have,



we have used the fact that the acceleration of block 2 is entirely in the x-direction, since it doesn’t move in the y-direction at all. Substituting these last two equations into the first yields,



and then solving for a yields,

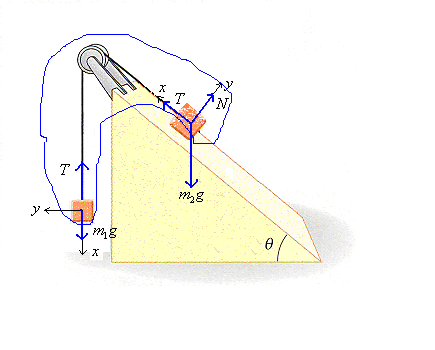


If we fill in our values, we get,



The positive sign indicates that the blocks are accelerating in the positive x-direction, which implies that m2 is going up the plane, and m­1 is going down the plane. If we had obtained a negative value for a, then this would mean m1­ would be going up the plane, and m2 down the plane.

Now let’s look at the example another way. Let’s look at the forces acting on the two blocks as a whole. Now draw (at least in our heads) a circle around are ‘object’ – the two blocks together.



Next we write out N2L for our object. Again the tension, T, is an internal force acting *between* our object, not *on* it. So we don’t include it.



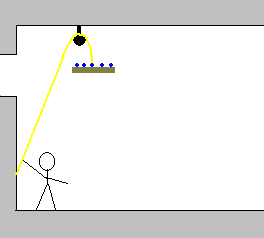
We can use the x-equation to solve for a. And we have,



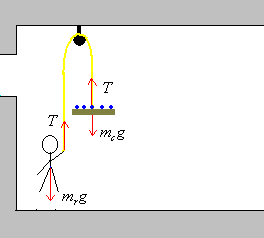
just like last time – only a lot faster.

**Example: Chandelier problem**

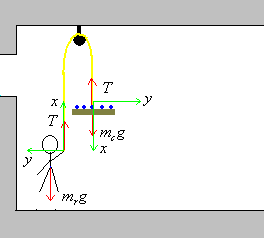
Suppose a chandelier is connected to a wall via pulleys. The chandelier has a mass of 100kg, and Russel Crowe has a mass of 80kg. If he cuts the cord holding the chandelier to the wall, the chandelier will fall, raising him up in the process. If the window is 15m high, how long will it take for him to reach it?



To answer the question, we’ll draw Russel Crowe and the chandelier and all of the forces acting on them during the process.



Then we write out N2L, but before we do we’ll draw in axes that help to analyze the problem most perspicuously. In such coupled motion problems, we always want to draw the axis so that the x-axis points along the rope, in the direction that the objects are moving if possible. So we draw our axes like this:



Now we’re ready to write N2L. For Russel Crowe we have:



and for the chandelier,



Plugging the first equation into the second we get,



Now then, to figure out how long it will take to reach the top, we’ll use a kinematics equation:



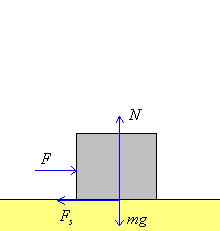
**Question 6**. What force F would you have to exert with each finger to hold a 6kg book. Suppose that the coefficient of static friction between your fingers and the book is μs = 1.3.





**Example: Static Friction force**

Suppose we have a box with mass m = 100kg resting on a surface with coefficient of static friction μs = 0.85. If we exert a force of 500N on it, will it move?



To determine whether it will move or not, we have to see whether F is greater than the maximum static friction force. Fsmax is given by:



We can get N from applying N2L in the vertical direction,



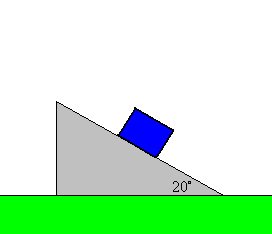
So then



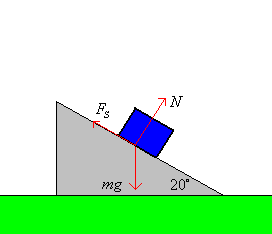
So the applied force is too weak. The box will not move. We would need to apply at least an 833N force to make the box move.

**Example: Static force of friction**

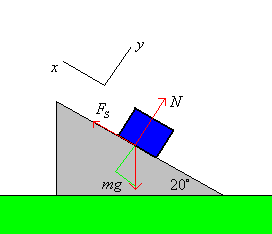
Consider the box, weight 350N, on the incline plane. If it isn’t moving, what is Fs?



First we label the forces,

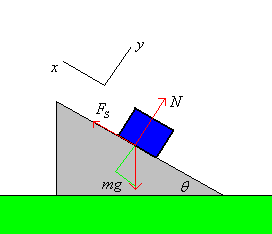


The direction of Fs is up towards the left since gravity would tend to pull the block down, and the static friction force opposes the tendency of the block to move downward. Now we can solve for Fs by breaking the gravitational force into its components in the x-y directions.





If the coefficient of static friction between the block and plane is 0.67, then at what angle would the plane have to be raised, in order for the block to move on its own?



At the requisite angle, the static friction force will be at its maximum value. In this case we’ll have,



and so we have,

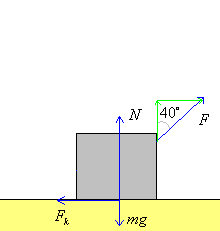


This is, incidentally, a good way to determine the coefficient of static friction between two surfaces.

**Example: pulling box along the ground**

Suppose you pull a box along the ground with a force of 500N directed along an angle of 40˚ degrees with respect to the vertical. If the box has a mass of 50kg, and a coefficient of kinetic friction μk = 0.3, what will be the box’s acceleration? How fast will it be moving after 7s?

The forces we have are drawn below. The horizontal and vertical components of the force, **F**, are shown in green.



We can determine the acceleration of the block using N2L,



using the fact that the acceleration is purely in the x-direction. Now equating the x-components we have,



but we don’t know N. So going back to N2L and equating the y-components we get,



substituting this into our expression for the acceleration we get,

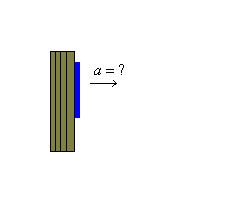


So this is our acceleration. Now, after 7s, the box’s velocity will be (assuming it started from rest)

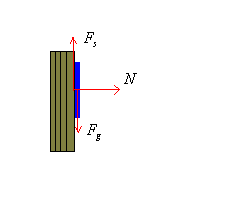


**Example: Keeping a pen from sliding vertically**

Suppose you have a pen with mass m = 25g. And suppose the coefficient of static friction between your pen and a book is 0.8. What horizontal acceleration must you give the book/pen to keep the pen from falling in the situation below?



If you are pushing the book to the right, then the forces acting on the pen will be these. There will be a normal fore from the book to the pen acting to the right. Gravity will act down, and the static friction force will act up, opposing the pen’s tendency to slide down.



According to N2L, and assuming the pen isn’t sliding vertically we have:



Now the minimum acceleration necessary is the one for which Fs is at its maximum value, i.e., Fsmax = μsN. So filling this in:



Solving for N, and plugging into the x-equation to get amin we find:



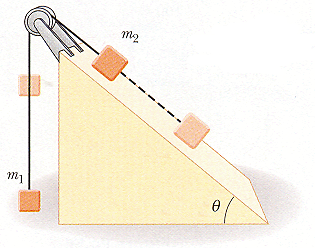
So the acceleration must be:



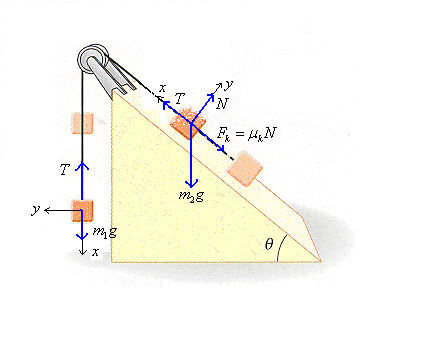
Interesting how the acceleration doesn’t depend on the mass of the pen.

**Example: Inclined plane with friction**

Consider two masses attached to an inclined. Suppose m1 = 10kg, and m2 = 20kg, and that the plane is inclined 30 degrees. And suppose that μk = 0.4. If we pull down on block 1 and then let go, what will be its acceleration?



To determine the acceleration we look at the forces acting on each object, and we orient our x-y axis for each block so that the x-axis points along the plane of motion of the objects.



Note that the friction force points opposite to the motion of the objects. Since we pulled on m1 to get it moving down, m2 will be moving up the plane, and therefore the friction force on m2 will point down the plane. Next we write out N2L for each object. For block 1 we have,



and for block 2 we have,



we have used the fact that the acceleration of block 2 is entirely in the x-direction, since it doesn’t move in the y-direction at all. Using the expression for N reduces the **x**-equation to:



and filling this into the top equation for block 1 gives,



Filling in our values, we have:

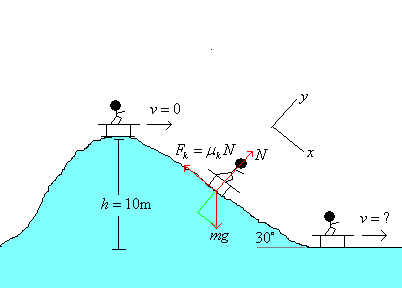


Since the acceleration is negative, this is indicative of the fact that the blocks will accelerate down the plane (or up in the case of m1). This means that the blocks will accelerate opposite to the direction of the initial velocity and so will slow down. If we want to know the tension in the ropes, then we can use the equation for block 1,



**Example: Sled ride**

Suppose you ride a sled down a hill 10m tall with a 30 degree incline. Suppose the coefficient of kinetic friction between the sled and snow of 0.2. And suppose your mass, along with the sled is 75kg. What will be your speed at the bottom?



Again, label the forces. Writing out Newton’s second law to determine the acceleration, which will be along the x-direction.



This breaks into the following equations,

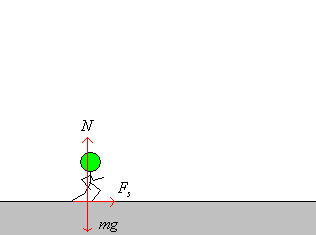


Now to determine the the velocity at the bottom, we use the relation,



**Example: How fast can you accelerate on ice?**

Suppose you’re standing on an ice rink, when you decide to run after someone skating by. How quickly can you accelerate if your mass is 60kg, and the coefficient of static fraction between your shoes and the ice is 0.15?



when you try to accelerate, your foot pushes on the ground, which pushes back on your foot. Since your foot is not sliding past the ground during this operation, the frictional force which the ground exerts on you is Fs. The maximum possible force that this can be is μsN. And this would correspond to your maximum possible acceleration. Applying N2L then,

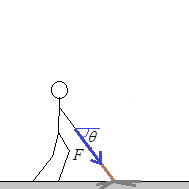


which is,

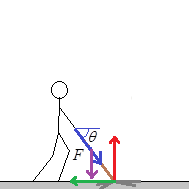


If you try to go any faster, the ground won’t be able to push back with the requisite force and your foot will just slip/slide past the ground. The same phenomenon is responsible for tires skidding when trying to accelerate too fast out of a stop light – and for pulling tablecloths out from under dinnerware.

7. You are pushing a mop as shown below. If the mop’s mass is m = 7kg, and the coefficient of static friction between mop and floor is μs = 0.85, what force F directed at angle θ = 35° will you need to apply to get the mop to move?



Labeling forces…



(red is normal force FN, green is the static friction force Fs, and purple is gravity Fg). Before writing out N2L we will need to determine the components of the applied force **F**. We can write it in vector form as

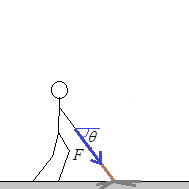
**F** = (Fcosθ, -Fsinθ) = (Fcos35, -Fsin35) = (0.82F, -0.57F). So Fx = 0.82F and Fy = -0.57F. Now we will write out N2L

 and 

Note that we have used Fs = μsFN b/c we are looking for the point where the mop is about to slip, and at this point (and only at this point) Fs will be at its maximum value. Now we will plug FN on the right into the equation on the left:



8. You are pushing the same mop across the floor with a force F = 50N. If the mop’s mass is m = 7kg, and the coefficient of kinetic friction between mop and floor is μk = 0.4, what will be the mop’s velocity vx and position x as a function of time? You may assume that vx and x are initially zero.



We will repeat the analysis, substituting Fk for Fs. So now we have:

 and 

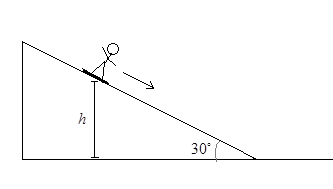
Plugging FN into the left equation to solve for ax we have:

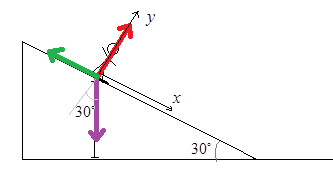


Plugging this into the kinematics equations we get:



9a. Suppose you’re skiing down a tall slope at the indicated angle of inclination. You want to start from a height sufficient to make your speed at the bottom be v = 27m/s. Let’s determine the required height. To start, assuming the coefficient of kinetic friction between the snow and your skis be μ­k = 0.2, write down an expression for vx and x as a function of time (you may take these to be 0 initially).





Drawing forces above, red is FN, purple is gravity, and green is FN. We have to vectorize gravity to determine its components in the x-y direction (remember we tilt the x-y axes to align with the plane). So then Fg = (mgsin30, -mgcos30). So Fgx = mgsin30 and Fgy = -mgcos30. N2L in the x and y directions is:

 and 

Plugging FN into the ax equation we can get ax…



Now we can fill in the velocity and position equations:



9b. Set vx equal to the desired value and solve for t, and then get x from it.

We want vx = 27 so we say:



and now plug this time into the x equation:



9c. Using x, solve for h.

So we have to be 114m up the slope. Using a little trig, we can say that h = 114sin30 = 57m. So we must be 72m above the ground.

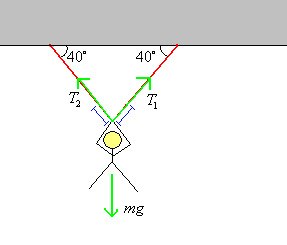
**Example**

Suppose you pull back at bow and arrow through a distance of 90cm, and that this requires a force of 500N. What is the spring constant of the bow and arrow?



**Example**

Suppose you’re a gymnast (m = 50kg) hanging onto two elastic ropes (k = 100N/m) as shown.



How much is each rope stretching (illustrated in blue) beyond its equilibrium length? To determine this, we draw the forces acting on the person and set the sum of forces equal to 0.



The first equation says that T1 = T2 and plugging this into the second equation yields,

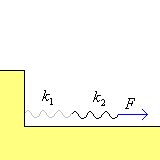


which is also the tension in T2 therefore. In order for the elastic ropes to have this tension they must each stretch an amount d such that,



**Example: adding springs in series**

What is the effective spring constant of two springs connected end to end (i.e., in series)? In other words, how is the force F, proportional to the displacement of the composite spring, Δx?



If we pull on the second spring with a force, F, then we will consequently pull on the first with that same force. The stretch of spring 1 would be according to:



and of spring 2,



so the net stretch would be:



which can be rearranged to say,



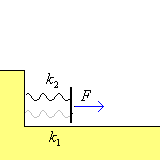
So the effective spring constant would be:



This formula will be seen again in the context of adding resistors in parallel.

**Example: adding springs in parallel**

What is the effective spring constant of two springs connected side by side?



In that case, the according to N2L,



So the effective spring constant is just the sum of the two spring constants,



**Example**

Suppose we attach a 2kg mass to a spring k = 10N/m, and stretch it out to 50cm beyond its equilibrium length. What will its position be as a function of time? What will be its maximum velocity?

Well, ω = √(k/m) = 2.24. A will be 0.5m, since we’re releasing it from rest there,

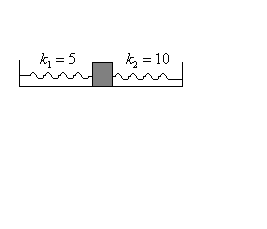
.So we’ll have,



The period will be T = 2π/ω = 2.81s.

The maximum velocity will be vmax = Aω = 1.12 m/s.

5. Suppose a mass is connected to two springs, as shown below. The spring constants are in N/m. The springs are at their equilibrium lengths. If you move the block to the right a distance of 2m, what will be the net force on it?



By displacing the block to the right 2m, spring 1 will exert a force to the left, i.e.,



and spring 2 will exert a force to the left as well,



The net force is:



6. Suppose you have to exert a force of 200N to stretch a spring 20cm. What force would you have to exert to stretch it 35cm?

From the first statement we can get the spring constant. The magnitude of the spring force is related to the distance stretched/compressed according to:

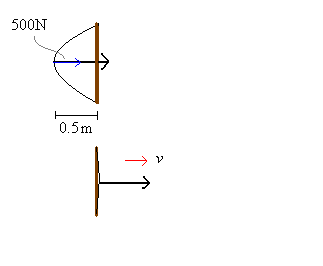


In order to stretch the spring 35cm then, the force will be:



**Example**

Suppose it take a force of 500N to stretch a bow 0.5m. If you fit a 100g arrow to it, how long will it take for the arrow to be released?



Since the arrow will be under the influence of a spring force until it gets past the bow, it will move sinusoidally until then as well. So up till then we can use the equations above. Going from x = -A to the equilibrium point corresponds to a quarter of a period. So Δt will be one quarter of a period. Since k = 500/0.5 = 1000N/m, and m = 0.1kg, ω0 = √1000 = 31.6s-1. Therefore, the period is T = 2π/ω0 = 0.2s. And therefore the time will be:



**Example**

When you (m = 70kg) jump into your car, you notice that it oscillates back and forth with a frequency of 2Hz. What is the spring constant of the shock absorbers in the car?

Well f = 2Hz → T = 1/f = 0.5s. Solving for k, we have,



which implies,



7. In an electric shaver, the blade moves back and forth over a distance of 2 mm in simple harmonic motion, with frequency 125 Hz. Find **(a)** the amplitude (in mm), **(b)** the maximum blade speed, and **(c)** the magnitude of the maximum blade acceleration.

The amplitude is ½ the total distance from ‘peak’ to ‘trough’. So this is A = 1mm. The maximum blade speed is vmax = Aω. Now ω = 2π/T, which is 2πf, which is 2π(125) = 785 s-1. So we get vmax = (0.001m)(785 s-1) = 0.785m/s. The maximum acceleration is:

amax = Aω2 = (0.001m)(785 s-1)2 = 616 m/s2.

1. Suppose we attach a 2kg mass to a spring k = 10N/m, and stretch it out to 50cm beyond its equilibrium length. What will be its maximum velocity?

Well, ω = √(k/m) = 2.24. A will be 0.5m, since we’re releasing it from rest there,

.So we’ll have,



Its velocity as a function of time will be:



and so the maximum velocity will be:



13. Suppose we attach a 2kg mass to a spring k = 10N/m, and stretch it out to 5m and then give it an initial velocity of 13m/s in the positive x direction. Where will it be after 2s? What will be its velocity? What will be its acceleration? What will be its period of motion, amplitude, maximum velocity, and maximum acceleration?

Going back to Newton’s second law, we found:



I’m now going to solve the equation again b/c I don’t feel like looking up what I wrote. But you can ignore this part if you wish. The general solution to this equation is:



Filling in the initial conditions that x(0) = x0 and vx(0) = v0 we have:



and,



Solving for A and φ we get:



So our solution is:



Now you can start paying attention again. Filling in the numbers we have:



Note that last number is in radians! Filling this in, we have:



So its position at t = 2s will be:



It velocity is given by:



and so its velocity at t = 2 will be:



And its acceleration as a function of time will be:



Therefore at time t = 2, its acceleration will be:



Its period is T = 2π/ ω = 2π/2.24 = 2.8s. Looking at the general formula for x(t), v(t), and a(t):



we see the amplitude is 7.66m, the maximum velocity is 17.2m/s, and the maximum acceleration is 38.5m/s2.

**10**. Suppose you’re oscillating back and forth on a bungee cord. If the amplitude of your oscillations is 5m, and your frequency of oscillation is *f* = 0.3Hz, what is your maximum speed during these oscillations?

Your maximum speed is given by vmax = Aω. ω = 2πf = 2π(0.3) = 1.89 s-1. So multiplying the two:



2. After a daring bungee jump into a 200m canyon, you attempt to determine the coefficient of the air resistance force, as well as the spring constant of the bungee cord. To that end, as you hang there oscillating back and forth, you estimate that your amplitude of oscillation decreases to half its initial value in 45s, and during this same time you have completed 5 oscillations. So what are k and b? You can take your mass to be 70kg.

the amplitude as a function of time is given by A(t) = Ae-γt, and so:

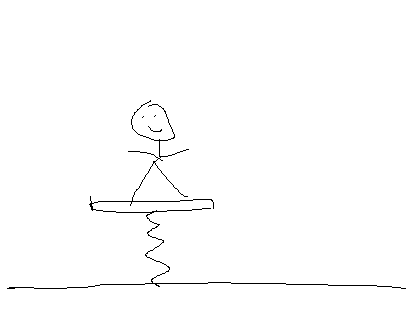


And so this means that b = 2mγ = 2(70)(0.0154) = 2.16N∙s/m. Then as far as k is concerned we can say:



**Example**

Suppose you are standing on an oscillating platform of mass m, whereas your mass is myou. And let the spring constant be k, and the amplitude A. What is the maximum amplitude of oscillation the spring can have before you will start to fly off of the spring?



Well assuming you’re on the platform, your acceleration will be:



where ω0 = √[k/(m+myou)]. And we must also have:



You will start to fly off if FN < 0, which would imply,

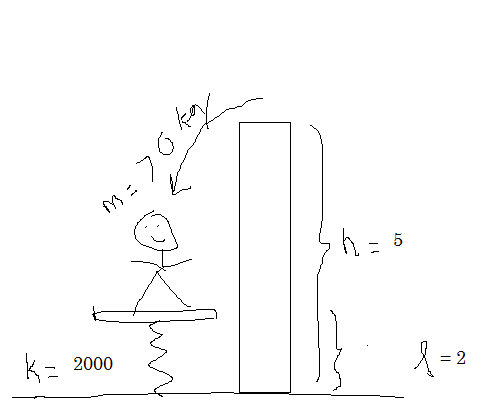


And so if g < Aω02, then you will eventually fly off. And this requires:



**Example**

Can replicate example above, but have you jump off of a building and determine what max height you could jump off of so that you stay on the spring at all subsequent times. Or what mass m of platform you would need to keep person from popping off.



Well, spring position will be x(t) = xeq. + Acos(ωt+φ0). Supposing mass of platform is small, zero. Then xeq. = … ΣF = 0 → -k(xeq.-ℓ) – mg = 0 → xeq. = ℓ - mg/k = 2 – (70)(9.8)/2000 = 1.66m. And initial position of spring will be x0 = ℓ = 2, and initial velocity will be your velocity, which would be … ΔE = 0 → mgh – mgℓ = (1/2)mv2 → v = -√2g(h-ℓ) = -7.67m/s. Oscillation frequency will be: ω = √(k/m) = √(2000/70) = 5.35 rad/s. So then so far we have x(t) = xeq. + Acos(ωt+φ0) = 1.66 + Acos(5.35t + φ0). Then applying initial conditions:



and,



These imply the amplitude is given by A = √(0.342 + 1.442) = 1.48m. And phase constant is given by φ0 = tan-1(1.44/0.34) = 1.34. So we have x(t) = 1.66 + 1.48cos(5.35t + 1.34).

**Example**

Suppose you (m = 70kg) step of canyon will bungee cord tied to your leg. The length of the cord is ℓ = 60m, and the spring constant is k = 20N/m. What will be your position as a function of time, measured as soon as the cord has unraveled to its full length? To what height will he rise when rebounding upward? To what maximum depth will he descend? What is period of oscillation?

Initial position is x­0 = 60. Initial velocity is v = √2g(60) = 34.3m/s. Equilibrium position is xeq. = ℓ + mg/k = Oscillation frequency is ω = √(k/m) = 0.535rad/s. So x(t) = 94.3m. So x(t) = 94.3 + Acos(0.535t+φ0). Applying initial conditions: x0 = xeq. + Acos(ωt + φ0) → 60 = 94.3 + Acos(0.535t + φ0) → -34.3 = Acos(0.535t + φ0); and v0 = -Aωsin(ωt + φ0) → 34.3 = 0.535Asin(ωt + φ0) → 64.2 = Asin(ωt + φ0). So A = 72.8m, and φ0 = tan-1(64.2/-34.3) = π – tan-1(64.2/34.3) = 2.06. So x(t) = 94.3 + 72.8cos(0.535t + 2.06). So he will rise to height of 94.3 – 72.8 = 21.5m below the platform. And he’ll descend to distance 167m below platform. Period of oscillation is 11.8s.

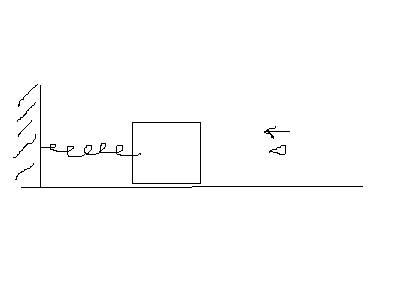
**Example**

Suppose car (m = 1500kg) has 4 shock absorbers with total spring constant k = 40 000N/m. And you (m = 70kg) sit on the car. What will be the height of the car, measured w/r to its initial position, as a function of time thereafter?

Well x = xeq. + Acos(ωt + φ0), where xeq. = ℓ - (myou + mcar)g/k = ℓ - 0.385, and ω = 5.05rad/s. So xeq. = ℓ - 0.385 + Acos(5.05t + φ0). Initial conditions are x0 = ℓ - mcarg/k = ℓ - 0.368, and v0 = 0. So we have: (1) ℓ - 0.368 = ℓ - 0.385 + Acos(5.05t+φ0) → 0.017 = Acos(φ0) and (2) 0 = 5.05Asin(φ0) → φ0 = 0. So A = 0.017. And therefore x(t) = (ℓ-0.385) + 0.017cos(5.05t). And so then the position w/r to the original position is x(t) – (ℓ - 0.368) = -0.17 + 0.17cos(5.05t).

**Example**

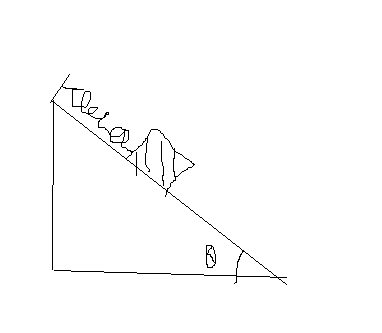
Suppose a block m = 2kg is connected to spring k = 15N/m. You shoot the block with bullet m = 0.020kg and velocity v = 800m/s. If the bullet embeds in the block instantaneously, what will be the position of the block as function of time, as measured w/r to it’s initial position?



x(t) = xeq. + Acos(ωt + φ0) = ℓ + Acos(ωt + φ0). Now ω = 2.73, ℓ = ?, x0 = ℓ, and v0 = (0.02)(800)/(2 + 0.02) = (-)7.92m/s. So x(t) = ℓ + Acos(2.73t + φ0). And (1) ℓ = ℓ + Acos(φ0) → 0 = Acos(φ0). And (2) -7.92 = -A(2.73)sin(φ0) → 2.9 = Asin(φ0). So these imply A = 2.9, and φ0 = π/2. So x(t) = ℓ + 2.9cos(2.73t + π/2). And relative to its initial position we’d have x(t) = 2.9cos(2.73t + π/2).

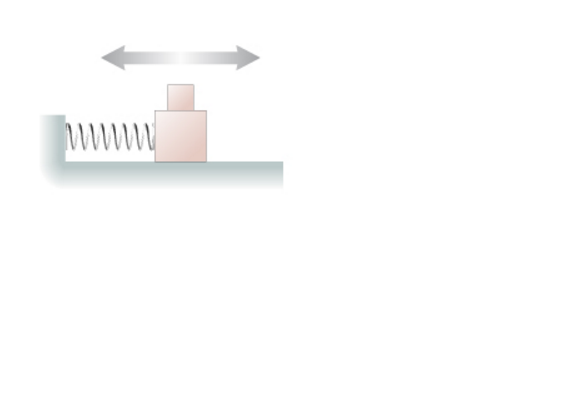
**Example**

A spring k is connected to a block m and rests on an incline plane at angle θ. If you perturb the block, what will be the period of oscillation?



N2L would read: -k(x-ℓ) + mgsinθ = mdx2/dt2. Period of oscillation is same.

**Question 1.** The two blocks shown below oscillate with frequency f = 0.67Hz. When the amplitude is increased to 18cm, the top block just begins to slip. What is the coefficient of static friction between the two blocks?



Generally, the motion of the blocks is given by x(t) = Acos(wt+φ0). And acceleration is a(t) = d2x/dt2 = -Aω2cos(ωt+φ0). So amax = Aω2 = A(2πf)2 = 4π2Af2. If the top block is about to slip, that means that the force responsible for its motion – the static friction force – has reached its maximum possible value μsFN = μsmg. And so from N2L we have μsmg = m[4π2Af2] 🡪 μs = 4π2Af2/g = 0.326.

**Example**

You sit in car and it depresses by d. Then you jump in car and it oscillates with frequency f. What is spring constant of shocks and mass of car?

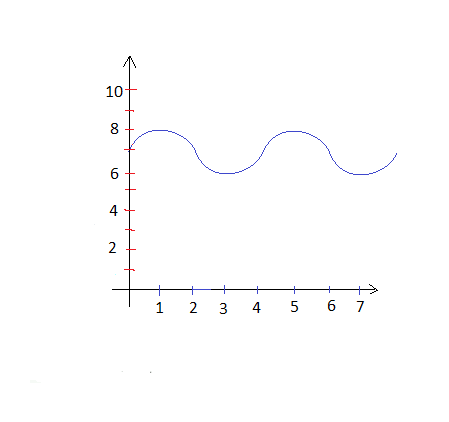


and then frequency of oscillation is:



so solve for mcar.

**Problem 5**. A mass on a spring oscillates according to the following diagram. What will be the mass’s speed at t = 3.5s?



Could say that A = 1m, ω = 2π/T = 2π/4 = π/2, and φ0 = - π/2. And so x(t) = Acos(ωt+φ0) = (1)cos(π/2∙t – π/2). Then the velocity would be: v(t) = -Aωsin(ωt + φ0). Filling in t = 3.5s, we’d have: 1.1m/s.

2. After a daring bungee jump into a 200m canyon, you attempt to determine the coefficient of the air resistance force, as well as the spring constant of the bungee cord. To that end, as you hang there oscillating back and forth, you estimate that your amplitude of oscillation decreases to half its initial value in 45s, and during this same time you have completed 5 oscillations. So what are k and b? You can take your mass to be 70kg.

the amplitude as a function of time is given by A(t) = Ae-γt, and so:



And so this means that b = 2mγ = 2(70)(0.0154) = 2.16N∙s/m. Then as far as k is concerned we can say:

